Proc 2nd Saudi Sci. Conf. Fac. Sci., KAU, 15-17 March 2004, Part II, pp. 73-80 (2005 A.D./1425 A.H.)

Some Properties of Copper – Aluminium Cu-Al At Different % Of Al

FAE'Q A. A. RADWAN
Faculty of Engineering, Near East University, K.K.T.C.
Lefkosa P.O. Box: 670, Mersin – 10, TURKEY
faeq@neu.edu.tr

ABSTRACT. The decomposition of elastic compliance tensor into its irreducible parts is given. The norm concept of elastic compliance tensor and its irreducible parts and their ratios are used to study the anisotropy of copper – aluminium at different % of Al, and the relationship of their structural properties and other properties with their anisotropy are given.

Key Words: Properties, Decomposition, Norm, Anisotropy, and Elastic Compliance.

Elastic Compliance Tensor Decomposition

The constitutive relation characterizing linear anisotropic solids is the generalized Hook's law [1]:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl},\ \varepsilon_{ij} = S_{ijkl}\sigma_{kl}, \tag{1}$$

Where σ_{ij} and ε_{kl} are the symmetric second rank stress and strain tensors, respectively, C_{ijkl} is the fourth-rank elastic stiffness tensor (elastic constant tensor) and S_{ijkl} is the elastic compliance tensor.

There are three index symmetry restrictions on these tensors. These conditions are:

$$S_{ijkl} = S_{jikl},\ S_{ijkl} = S_{ijlk},\ S_{ijkl} = S_{klij}, \tag{2}$$

Where the first equality comes from the symmetry of stress tensor, the second one from the symmetry of strain tensor, and the third one is due to the presence of a deformation potential. In general, a fourth-rank tensor has 81 elements. The index symmetry conditions (2) reduce this number to 21. Consequently, for most asymmetric materials (triclinic symmetry) the elastic constant tensor has 21 independent components.

Elastic constant tensor C_{ijkl} possesses the same symmetry properties as the elastic constant tensor S_{ijkl} and their connection is given by :

$$C_{ijkl}S_{klmn}=\frac{1}{2}\left(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm}\right), \text{ Where } \delta_{ij} \text{ is the Kronecker delta} \quad (3)$$

The Einstein summation convention over repeated indices is used and indices run from 1 to 3 unless otherwise stated.

Schouten [3] has shown that C_{ijkl} can be decomposed into two scalars, two deviators, and one-nonor parts.

By applying the symmetry conditions (2) to the decomposition results obtained for a general fourth-rank tensor, the following reduction spectrum for the elastic compliance tensor is obtained. It contains two scalars, two deviators, and one-nonor parts [4, 5, and 6]:

$$S_{ijkl}=S_{ijkl}^{(0;1)}+S_{ijkl}^{(0;2)}+S_{ijkl}^{(2;1)}+S_{ijkl}^{(2;2)}+S_{ijkl}^{(4;1)}. \quad (4)$$

Where

$$S_{ijkl}^{(0;1)}=\frac{1}{9}\delta_{ij}\delta_{kl}S_{ppqq}. \quad (5)$$

$$S_{ijkl}^{(0;2)}=\frac{1}{90}\left(3\delta_{ik}\delta_{jl}+3\delta_{il}\delta_{jk}-2\delta_{ij}\delta_{kl}\right)\left(3S_{pqpq}-S_{ppqq}\right). \quad (6)$$

$$S_{ijkl}^{(2;1)}=\frac{1}{5}\left(\delta_{ik}S_{jplp}+\delta_{jk}S_{iplp}+\delta_{il}S_{jpkp}+\delta_{jl}S_{ipkp}\right) -\frac{2}{15}\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right)S_{pqpq} \quad (7)$$

$$S_{ijkl}^{(2;2)}=\frac{1}{7}\delta_{ij}\left(5S_{klpp}-4S_{kplp}\right)+\frac{1}{7}\delta_{kl}\left(5S_{ijpp}-4S_{ipjp}\right) -\frac{2}{35}\delta_{ik}\left(5S_{jlpp}-4S_{jplp}\right)-\frac{2}{35}\delta_{jl}\left(5S_{ikpp}-4S_{ipkp}\right) -\frac{2}{35}\delta_{il}\left(5S_{jkpp}-4S_{jpkp}\right)-\frac{2}{35}\delta_{jk}\left(5S_{ilpp}-4S_{iplp}\right) +\frac{2}{105}\left(2\delta_{jk}\delta_{il}+2\delta_{ik}\delta_{jl}-5\delta_{ij}\delta_{kl}\right)\left(5S_{ppqq}-4S_{pqpq}\right). \quad (8)$$

$$S_{ijkl}^{(4;1)} = \frac{1}{3}(S_{ijkl} + S_{ikjl} + S_{iljk}) - \frac{1}{21}\left[\delta_{ij}\left(S_{klpp} + 2S_{kplp}\right) + \delta_{ik}\left(S_{jlpp} + 2S_{jplp}\right)\right.$$

$$+ \delta_{il}\left(S_{jkpp} + 2S_{jpkp}\right) + \delta_{jk}\left(S_{ilpp} + 2S_{iplp}\right) + \delta_{jl}\left(S_{ikpp} + 2S_{ipkp}\right)$$

$$\left. + \delta_{kl}\left(S_{ijpp} + 2S_{ipjp}\right)\ \right] + \frac{1}{105}\left[\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\left(S_{ppqq} + 2S_{pqpq}\right)\ \right] \quad ...(9)$$

These parts are orthonormal to each other. Using Voigt's notation [1] C_{ijkl} and S_{ijkl}, can be expressed in 6 by 6 reduced matrix notation, where the matrix coefficients $c_{\mu\lambda}$ and s_{mn} are connected with the tensor components C_{ijkl} and S_{ijkl} respectively by the recalculation rules:

$$C_{ijkl} = c_{\mu\lambda} \qquad (ij \leftrightarrow \mu = 1,....,6, kl \leftrightarrow \lambda = 1,....,6);$$

That is:

$$11 \leftrightarrow 1,\ 22 \leftrightarrow 2,\ 33 \leftrightarrow 3,\ 23 = 32 \leftrightarrow 4,\ 31 = 13 \leftrightarrow 5,\ 12 = 21 \leftrightarrow 6;$$

And

$S_{ijkl} = s_{mn}$ When m and n are 1, 2 or 3,

$2S_{ijkl} = s_{mn}$ When either m or n are 4, 5 or 6,

$4S_{ijkl} = s_{mn}$ When both m and n are 4, 5 or 6.

The Norm Concept

Generalizing the concept of the modulus of a vector, norm of a Cartesian tensor (or the modulus of a tensor) is defined as the square root of the contracted product over all indices with itself:

$$N = \|T\| = \left\{T_{ijkl.........}.T_{ijkl.........}\right\}^{1/2}$$

Denoting rank-n Cartesian $T_{ijkl.........}$, by T_n, the square of the norm is expressed as [7]:

$$N^2 = \|T\|^2 = \sum_{j,q}\left\|T^{(j;q)}\right\|^2 = \sum_{(n)} T_{(n)}T_{(n)} = \sum_{(n),j,q} T_{(n)}^{(j;q)}T_{(n)}^{(j,q)}.$$

This definition is consistent with the reduction of the tensor in Cartesian formulation when all the irreducible parts are embedded in the original rank-n tensor space.

Since the norm of a Cartesian tensor is an invariant quantity, we suggest the following:

Rule1. The norm of a Cartesian tensor may be used as a criterion for representing and comparing the overall effect of a certain property of the same or different symmetry. The larger tbe norm value, the more effective the property is.

It is known that the anisotropy of the materials, i.e., the symmetry group of the material and the anisotropy of the measured property depicted in the same materials may be quite different. Obviously, the property, tensor must show, at least, the symmetry of the material. For example, a property, which is measured in a material, can almost be isotropic but the material symmetry group itself may have very few symmetry elements. We know that, for isotropic materials, the elastic compliance tensor has two irreducible parts, i.e., two scalar parts, so the norm of the elastic compliance tensor for isotropic materials depends only on the norm of the scalar parts, i.e. $N = N_s$. Hence, the ratio $\frac{N_s}{N} = 1$ for isotropic materials. For anisotropic materials, the elastic constant tensor additionally contains two deviator parts and one nonor part, so we can define $\frac{N_d}{N}$ for the deviator irreducible parts and $\frac{N_n}{N}$ for nonor parts. Generalizing this to irreducible tensors up to rank four, we can define the following norm ratios: $\frac{N_s}{N}$ for scalar parts, $\frac{N_v}{N}$ for vector parts, $\frac{N_d}{N}$ for deviator parts, $\frac{N_{se}}{N}$ for septor parts, and $\frac{N_n}{N}$ for nonor parts. Norm ratios of different irreducible parts represent the anisotropy of that particular irreducible part, they can also be used to asses the anisotropy degree of a material property as a whole. We suggest the following two more rules:

Rule 2. When N_s is dominating among norms of irreducible parts: the closer the norm ratio $\frac{N_s}{N}$ is to one, the closer the material property is isotropic.

Rule3. When N_s is not dominating or not present, norms of the other irreducible parts can be used as a criterion. But in this case the situation is reverse; the larger the norm ratio value we have, the more anisotropic the material property is.

The square of the norm of the elastic compliance tensor S_{mn} is:

$$\|N\|^2 = \sum_{mn}\left(S_{mn}^{(0;1)}\right)^2 + \sum_{mn}\left(S_{mn}^{(0;2)}\right)^2 + 2\sum_{m,n}\left(S_{mn}^{(0;1)}.S_{mn}^{(0;2)}\right) + \sum_{mn}\left(S_{mn}^{(2;1)}\right)^2 + \sum_{mn}\left(S_{mn}^{(2;2)}\right)^2 + 2\sum_{mn}\left(S_{mn}^{(2;1)}.S_{mn}^{(2;2)}\right) + \sum_{mn}\left(S_{mn}^{(4;1)}\right)^2 \quad (10)$$

Let us consider the irreducible decompositions of the elastic compliance tensor in the following crystals.

Table 1. Elastic Compliance $(TPa)^{-1}$ [8]

Element, Cubic System	s_{11}	s_{12}	s_{44}
Copper, Cu	15.00	-6.30	13.30
Aluminium, Al	16.00	-5.80	35.3

Table 2. Elastic Compliance $(TPa)^{-1}$ [8]

Alloy, Cubic System Copper Aluminium, Cu-Al, At % Al	s_{11}	s_{12}	s_{44}
0.04	14.87	-6.23	13.21
0.20	14.99	-6.28	13.21
0.75	14.99	-6.28	13.14
1.00	15.06	-6.32	13.19
1.95	15.08	-6.33	13.16
2.14	15.08	-6.33	13.12
2.21	15.21	-6.39	13.14
3.10	15.34	-6.45	13.12
4.00	15 53	-6.55	13.07
4 81	15.90	-6.73	13.35
4.85	15.56	-6.56	13.04
5.00	15.63	-659	13.06
6.50	15.86	-6.71	12.97
7.05	15.97	-6.76	12.99
7.50	15.90	-6.72	12.92
8.40	16.22	-6.88	12.90
9.00	16.55	-7.05	12.75
9.86	16.70	-7.11	12.82
10.22	16.69	-7.11	12.82
10.80	16.86	-7.19	12.81
11.77	17.13	-7.32	12.76
12.55	17.41	-7.46	12.67
13 25	17.62	-7.56	12.69
14.00	18.12	-7.78	12.45

By using table 1 and table 2, and the decomposition of the elastic compliance tensor, we calculated the norms and the norm ratios as in table 3 and in table 4.

Table 3, the norms and norm ratios

Element	N_s	N_d	N_n	N	N_s/N	N_d/N	N_n/N
Copper, Cu	46.8696	0	23.803	52.568	0.8916	0	0.4528
Aluminium, Al	72.3853	0	06.743	72.699	0.9957	0	0.0928

Table 4, the norms and norm ratios

Alloy, Cubic System Copper Alumiuium, Cu-Al, at % Al	N_s	N_d	N_n	N	N_s/N	N_d/N	N_n/N
0.04	46.4703	0	23.552	52.098	0.8920	0	0.4521
0.20	46.7254	0	23.828	52.450	0.8909	0	0.4543
0.75	46.6470	0	23.885	52.406	0.8901	0	0.4558
1.00	46.8669	0	24.023	52.665	0.8899	0	0.4561
1.95	46.8781	0	24.096	52.708	0.8894	0	0.4572
2.14	46.8333	0	24.128	52.683	0.8890	0	0.4580
2.21	47.1402	0	24.421	53.090	0.8879	0	0.4600
3.10	47.4023	0	24.746	53.473	0.8865	0	0.4628
4.00	47.7792	0	25.258	54.044	0.8841	0	0.4673
4.81	48.9157	0	25.924	55.361	0.8836	0	0.4683
4.85	47.8059	0	25.347	54.110	0.8835	0	0.4684
5.00	47.9783	0	25.493	54.331	0.8831	0	0.4692
6.50	48.4000	0	26.135	55.005	0.8799	0	0.4751
7.05	48.6621	0	26.379	55.352	0.8791	0	0.4766
7.50	48.4197	0	26.257	55.081	0.8791	0	0.4767
8.40	49.1148	0	27.053	56.073	0.8759	0	0.4825
9.00	49.6935	0	27.987	57.033	0.8713	0	0.4907
9.86	50.0874	0	28.272	57.516	0.8709	0	0.4915
10.22	50.0720	0	28.255	57.494	0.8709	0	0.4914
10.80	50.4349	0	28.670	58.014	0.8694	0	0.4942
11.77	50.9773	0	29.360	58.828	0.8666	0	0.4991
12.55	51.5043	0	30.116	59.663	0.8633	0	0.5048
13.25	51.9906	0	30.603	60.329	0.8618	0	0.5073
14.00	52.8008	0	31.968	61.724	0.8554	0	0.5179

Conclusiou

1) From table (3), considering the ratio $\frac{N_s}{N}$ we can say that Copper, (first ionization energy is 745KJ/mole) is more anisotropic than Aluminium, (first ionization energy is 577.9KJ/mole), and considering the value of N which is more bigb in the case of Aluminium, so we can say that Aluminium elastically is stronger than Copper.

2) From table (4) considering the ratio $\frac{N_s}{N}$ we can say that in the Alloy Cu-Al as the percentage of Al increases the anisotropy of the alloy increases, and considering the value of N which is increasing as the percentage of Al increases, so we can say that the alloy becomes elastically strongest.

References

[1] **Nye, J.F.** (1964), *"Physical Properties of Crystals, Their Representation by Tensors and Matrices"*, (Oxford University Press p131-149).

[2] **Teodosio, C.** (1982), *"Elastic Model of Crystal Defects"*, (Springer-Verlag, Berlin, p. 57).

[3] **Schouten, J.A.** (1954), *"Tensor Analysis for Physicists"*, (Clarenden Press, p. 157).

[4] **Radwan, Fae'q A.A.** (2001), *"Scalar Irreducible Parts of Sixth Rank Tensor"*, Arab Gulf Journal of Scientific Research, 19 (3), p163-166.

[5] **Radwan, Fae'q A.A.** (2000), *"Irreducible Parts of Elastic Compliance Tensor and Anisotropy"*, The International Union of Materials Research Societies-6th International Conference in Asia, IUMRS-ICA 2000, Multi-Scale Materials Modelling, 23-26 July 2000, City University of Hong Kong, Hong Kong.

[6] **Radwan, Fae'q A.A.** (1999), *"Norm Ratios and Anisotropy Degree"*, The First International Conference in Mathematical Sciences, Applied Mathematics, ICM99, Novemher 1999, United Arab Emirates University, Al Ain, United Arab Emirates.

[7] **Jerphagnou, J., Chemla, D.S.,** and **Bonnevile, R.,** (1978), *"The Description of Condensed Matter Using Irreducible Tensors"*, Adv. Phys, 11, p1003-1017.

[8] **Landolt-Börnstein, Group III,** *"Crystal and Solid State Physics"*, Volume, 11, Springer-Verlag.

بعض ملكيّات (خواص) النّحاس – الألومنيوم بنسب مئوية مختلفة من الألومنيو

فائق عبدالرحيم رضوان
كلية الهندسة ، جامعة الشرق الأدنى
نيقوسيا – جمهورية شمال قبرص

المستخلص. تم إعطاء تحلل عضلة شادة الطاعة المـرن (المطـاط) إلـى أجـزاء التخفيض (الأجزاء غير القابلة للانقاص،التي يصعب اختزالها. أعطى مفهوم المعيـار (المبدأ) لعضلة شادة الطاعة المرن (المطاط) وأجزائة غير القابلة للاختزال ونـسبهم، واستعملت لدراسة الأنيسوتروبي (ليست موحدة الخواص في جميع الاتجاهات، متباينة الخواص) للنحاس – الألومنيوم بنسب مئوية مختلفة من الألومنيوم وعلاقة ملكيَـاتهم (خواصهم) البنائيَة و الملكيَات (الخواص) الأخرى مع الأنيسوتروبي لهم.